

VARIANTA 3

I. a) $A + B = \begin{pmatrix} 2 & 2 \\ 1 & -1 \end{pmatrix}$; b) $x = \pm 1$; c) 3; d) $-\sqrt{2}$; e) $x = 3$;

f) $11 + 33 + 55 + 77 + 99 = 275$.

II. 1. a) {2,3}; b) $A - C = \{1\} = B - C$; c) 3 funcții; d) $n \in \{1,2\}$;

e) 111, 112, 121, 211, 122, 212, 221, 222.

2. a) 80; b) $\triangle ABC$ este dreptunghic în A, $h = \frac{30 \cdot 16}{34} = \frac{240}{17}$; c) $\frac{30 \cdot 16}{2} = 240$;

d) $\frac{34}{2} = 17$; e) $BD = \frac{450}{17}$.

III.

a) $P = 3l \Rightarrow l = 1$.

b) $A_E = \frac{l^2 \sqrt{3}}{4} = \frac{\sqrt{3}}{4}$.

c) $\begin{cases} 1 + c^2 = ip^2 \\ 1 + c + ip = 3 \end{cases} \Leftrightarrow \begin{cases} c = 2 - ip \\ 1 + (4 - 4ip + ip^2) = ip^2 \end{cases} \Leftrightarrow \begin{cases} c = 2 - ip \\ 4ip = 5 \end{cases}, ip = \frac{5}{4}; c = \frac{3}{4}$.

d) $A_U = \frac{\frac{3}{2}}{2} = \frac{3}{8}$.

e) $c_1 = c_2 = c \Rightarrow \begin{cases} 2c + ip = 3 \\ ip = c\sqrt{2} \end{cases} \Rightarrow c = \frac{6 - 3\sqrt{2}}{2}; ip = 3\sqrt{2} - 3$.

f) $A_I = \frac{\left(\frac{6 - 3\sqrt{2}}{2}\right)^2}{2} = \frac{27 - 18\sqrt{2}}{4}$.

g) $A_U < A_I < A_E$.

IV.

a) $\frac{1}{1} = 1 \in A$; pt $n=2, \frac{1}{2} \in A$; pt $n=3, \frac{1}{3} \in A$, pt $n=4, \frac{1}{4} \in A$.

b) $\frac{1}{n} \neq 2, \frac{1}{n} \neq \frac{3}{2} \forall n \in \mathbf{N}^*$, căci $\frac{1}{n} \leq 1, \forall n \in \mathbf{N}^*$.

c) Calcul direct.

d) $\left\{\frac{1}{2}\right\} \subset A \Rightarrow \frac{1}{2} \in B, \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right\} \subset A, 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 2 \Rightarrow 2 \in B$.

e) $\frac{1}{2^k + 1} + \frac{1}{2^k + 2} + \dots + \frac{1}{2^k + 2^k} > \frac{1}{2^k + 2^k} + \frac{1}{2^k + 2^k} + \dots + \frac{1}{2^k + 2^k} = \frac{2^k}{2^k + 2^k} = \frac{2^k}{2^{k+1}} = \frac{1}{2}$

f) Din e),

$$1 + \frac{1}{2} + \frac{1}{2+1} + \frac{1}{2+2} + \frac{1}{2^2+1} + \dots + \frac{1}{2^2+2^2} + \frac{1}{2^3+1} + \dots + \frac{1}{2^3+2^3} + \frac{1}{2^4+1} + \dots + \frac{1}{2^4+2^4} + \dots + \frac{1}{2^5+1} + \dots + \frac{1}{2^5+2^5} + \frac{1}{2^6+1} + \dots + \frac{1}{2^6+2^6} > 1 + \frac{1}{2} + \dots + \frac{1}{2} = 1 + \frac{7}{2}$$

$$\text{g) } 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2^{4014}} > 1 + \frac{2007 \cdot 2}{2} = 1 + 2007 = 2008 > 2007.$$

SNEE