Soluție

1.a) $\lim\limits_{x\to\infty} f(x) = +\infty$ și $\lim\limits_{x\to\infty} \frac{f(x)}{x} = +\infty$, deci *f* nu admite asimptotă spre $+\infty$.

b) $f'(x) = 0 \Leftrightarrow (n+1)x^n - (n+2) = 0$, deci avem punctul de extrem unic $x_n = \sqrt[n]{\frac{n+2}{n+1}}$.

c) $\lim\limits_{n\to\infty} x_n^{n^2} = \lim\limits_{n\to\infty} \left(1+\frac{1}{n+1}\right)^n = e$.

2.a) $I_1 = \int_0^1 \frac{x^2+1-1}{1+x^2}dx = \int_0^1 \left(1-\frac{1}{1+x^2}\right)dx = x\,\Big|_0^1 - arctg\left(x\right)\Big|_0^1 = 1-\frac{\pi}{4}$.

b) $I_{n+1} + I_n = \int_0^1 \frac{x^{2n+2}+x^{2n}}{1+x^2}dx = \int_0^1 x^{2n}dx = \frac{1}{2n+1}$.

c) $\frac{x^{2n}}{1+x^2} \le x^{2n},\ \ \forall x \in \left[0,1\right] \Rightarrow 0 < I_n \le \int_0^1 x^{2n}dx = \frac{1}{2n+1} \to 0$, deci $\lim\limits_{n\to\infty} I_n = 0$.