Soluții

1.a) $I_3+B=\begin{pmatrix}1&3&4\\0&1&3\\0&0&1\end{pmatrix}\Rightarrow \det(I_3+B)=\begin{vmatrix}1&3&4\\0&1&3\\0&0&1\end{vmatrix}=1$.

b)

$$f(A)=A^2-3A+I_3=\begin{pmatrix}3&1&1\\0&3&1\\0&0&3\end{pmatrix}\cdot\begin{pmatrix}3&1&1\\0&3&1\\0&0&3\end{pmatrix}-3\begin{pmatrix}3&1&1\\0&3&1\\0&0&3\end{pmatrix}+\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=\begin{pmatrix}9&6&7\\0&9&6\\0&0&9\end{pmatrix}-\begin{pmatrix}9&3&3\\0&9&3\\0&0&9\end{pmatrix}+\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=$$

$$=\begin{pmatrix}1&3&4\\0&1&3\\0&0&1\end{pmatrix}=I_3+B.$$

c) $\left(f(A)\right)^3=\left(I_3+B\right)^3=I_3^3+3I_3^2B+3I_3B^2+B^3=I_3+3B+3B^2+B^3$

$B^2=\begin{pmatrix}0&3&4\\0&0&3\\0&0&0\end{pmatrix}\cdot\begin{pmatrix}0&3&4\\0&0&3\\0&0&0\end{pmatrix}=\begin{pmatrix}0&0&9\\0&0&0\\0&0&0\end{pmatrix}$ şi $B^3=O_3$ de unde rezultă concluzia.

2.a) $x\circ x=x*x\Rightarrow (x-3)^2+3=2x-3\Rightarrow x^2-8x+15=0$ care are soluțiile $x_1=3$ şi $x_2=5$.

b) $x\circ a=3,\forall x\in\square\Leftrightarrow$

$(x-3)(a-3)+3=3,\forall x\in\square\Leftrightarrow$

$(x-3)(a-3)=0,\forall x\in\square\Rightarrow a=3\in\square$

c) $\begin{cases}x*(y+1)=4\\(x-y)\circ 1=5\end{cases}\Leftrightarrow\begin{cases}x+y=6\\(x-y-3)(-2)=2\end{cases}\Leftrightarrow\begin{cases}x+y=6\\-2x+2y=-4\end{cases}\Leftrightarrow\begin{cases}x+y=6\\-x+y=-2\end{cases}$ care are soluțiile intregi x=4 şi y=2.