

Soluție

1.a. $A \cdot B = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 5 & 4 \\ 3 & 1 \end{pmatrix} = \begin{pmatrix} 13 & 9 \\ 1 & -2 \end{pmatrix}.$

b. Notăm $X = \begin{pmatrix} x & y \\ z & t \end{pmatrix}$ ecuația devine $A \cdot X = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 3 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2x+z & 2y+t \\ -x+2z & -y+2t \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 3 & 1 \end{pmatrix} \Rightarrow X = \begin{pmatrix} \frac{7}{5} & \frac{7}{5} \\ \frac{11}{5} & \frac{6}{5} \end{pmatrix}.$

c. $A^2 = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} \Rightarrow A^2 - 4A + 5I_2 = \begin{pmatrix} 3 & 4 \\ -4 & 3 \end{pmatrix} - 4 \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} + 5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O_2.$

2. a. $x \circ x = 2 \Leftrightarrow 2x - 14 = 2 \Leftrightarrow x = 8.$

b. $(x \circ y) \circ z = (x + y - 14) \circ z = x + y + z - 28 = x \circ (y \circ z).$

c. asociativitatea din punctul **b**); elementul neutru este $e = 14$; elementele simetrizabile $x' = 28 - x \in \mathbb{R}, \forall x \in \mathbb{R}$;

$x \circ y = x + y - 14 = y + x - 14 = y \circ x.$