

SUBIECTUL al III-lea

(30 de puncte)

1. $\det A = \begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = 1 \cdot 2 - 2 \cdot 2 = 2 - 4 = -2$	3p 2p
2. $A \cdot \begin{pmatrix} -1 & 1 \\ 1 & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 \cdot (-1) + 2 \cdot 1 & 1 \cdot 1 + 2 \cdot \left(-\frac{1}{2}\right) \\ 2 \cdot (-1) + 2 \cdot 1 & 2 \cdot 1 + 2 \cdot \left(-\frac{1}{2}\right) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$ $\begin{pmatrix} -1 & 1 \\ 1 & -\frac{1}{2} \end{pmatrix} \cdot A = \begin{pmatrix} (-1) \cdot 1 + 1 \cdot 2 & (-1) \cdot 2 + 1 \cdot 2 \\ 1 \cdot 1 + \left(-\frac{1}{2}\right) \cdot 2 & 1 \cdot 2 + \left(-\frac{1}{2}\right) \cdot 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I_2$, deci matricea $\begin{pmatrix} -1 & 1 \\ 1 & -\frac{1}{2} \end{pmatrix}$ este inversa matricei A	2p 3p
3. $A \cdot A = \begin{pmatrix} 5 & 6 \\ 6 & 8 \end{pmatrix}, 3A = \begin{pmatrix} 3 & 6 \\ 6 & 6 \end{pmatrix}$ $A \cdot A - 3A = \begin{pmatrix} 5 & 6 \\ 6 & 8 \end{pmatrix} - \begin{pmatrix} 3 & 6 \\ 6 & 6 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2I_2$	3p 2p
4. $A - xI_2 = \begin{pmatrix} 1-x & 2 \\ 2 & 2-x \end{pmatrix} \Rightarrow \det(A - xI_2) = \begin{vmatrix} 1-x & 2 \\ 2 & 2-x \end{vmatrix} = x^2 - 3x - 2$ $x^2 - 3x - 2 = 2 \Leftrightarrow x^2 - 3x - 4 = 0 \Leftrightarrow x = -1 \text{ sau } x = 4$	3p 2p
5. $A \cdot A = 3A + 2I_2 \Rightarrow (A \cdot A) \cdot A = (3A + 2I_2) \cdot A = 3A \cdot A + 2A = 3(3A + 2I_2) + 2A = 11A + 6I_2$ Cum matricea A este nenulă, $11A + 6I_2 = aA + 6I_2 \Leftrightarrow a = 11$	3p 2p
6. $A \cdot X = \begin{pmatrix} 2+2p & 1+2q \\ 4+2p & 2+2q \end{pmatrix}, X \cdot A = \begin{pmatrix} 4 & 6 \\ p+2q & 2p+2q \end{pmatrix}$ Cum $\begin{pmatrix} 2+2p & 1+2q \\ 4+2p & 2+2q \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ p+2q & 2p+2q \end{pmatrix}$, obținem $p = 1$ și $q = \frac{5}{2}$	2p 3p